AP Problems: Applications of Derivatives (Possible Test Problems)

1. Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let y = f(x) be the particular solution to this differential equation with the initial condition f(1) = 0. For this particular solution, f(x) < 1 for all values of x.

b) Find $\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.

2. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$. Let y = g(x) be the particular solution to the given differential equation with g(-1) = 2.

a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

b) Find $\lim_{x \to -1} \left(\frac{g(x) - 2}{3(x+1)^2} \right)$

2. (calculator not allowed)

An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

5. (calculator not allowed)

An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point $(1, 5)$ is

1. Bob is riding his bicycle along a path $0 \le t \le 10$, Bob's velocity is modeled by
B(t) = $t^3 - 6t^2 + 300$, where t is measured in minutes and B(t) is measured in meters per minute.
Find Bob's acceleration at time t = 5.

2. Bob is riding his bicycle along a path $0 \le t \le 10$, Bob's velocity is modeled by
B(t) = $t^3 - 6t^2 + 300$, where t is measured in minutes and B(t) is measured in meters per minute.
Find Bob's average acceleration at time t = 5.

3. A curve C is defined by the parametric equations $x = t^2 - 4t + 1$ and $y = t^3$. Which of the following is an equation of the line tangent to the graph of C at the point (-3, 8)?

92. Let f be the function defined by $f(x) = x + \ln(x)$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over [1, 4]?

10. (calculator allowed)
The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

12. (calculator not allowed)

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$.

(Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.